Cellular automata and beyond

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Biological process

An environmental process whose actors are living organisms such as fungi, plants or animals.

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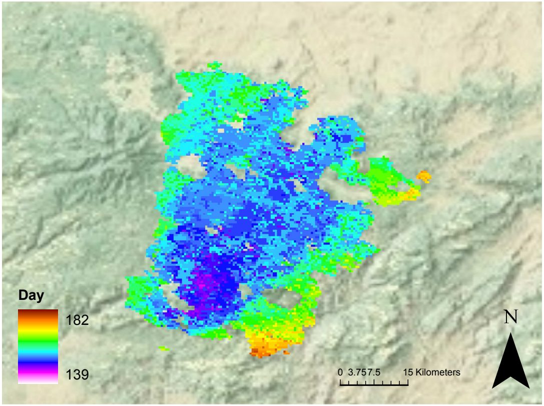

A logical need

- All processes taking place on earth are inherently characterized by both spatial and temporal dynamics

> *Each organic being is either directly or indirectly related in the most important manner to other organic beings.*
>
> *Darwin, 1859*

- Temporal models largely discard the intricate spatial dynamics by assuming homogeneity
- Nowadays, increasing availability of spatial observation techniques, yields
  1. information about the spatial structure of the medium in which the modelled processes occur
  2. spatio-temporal data enabling validation of a model

Spatial data

micro

meso

macro

Spatial data

micro

meso

macro

- confocal laser scanning microscopy
- magnetic resonance imaging
- spectrometry
- tomography

Spatial data

micro

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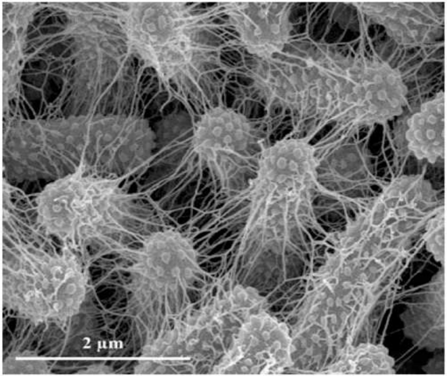
macro



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Spatial data

micro

meso

macro

- digital photography
- magnetic resonance imaging
- thermography

Spatial data

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meso

macro

Spatial data

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- aerial photography
- radar imaging
- satellite imaging

Spatial data

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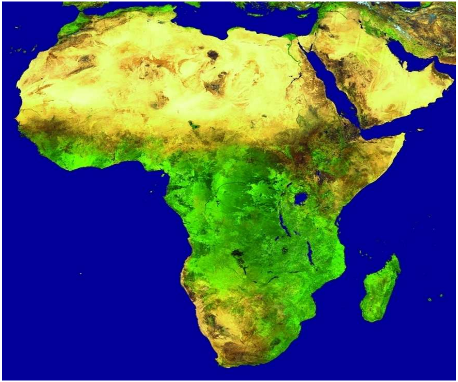
meso

macro



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Mathematical models of environmental processes

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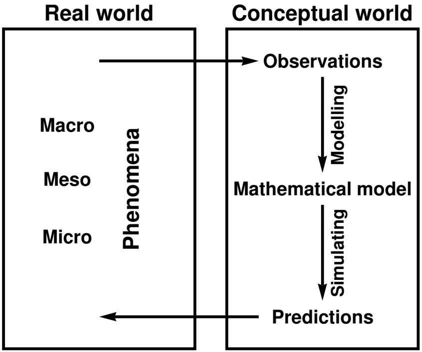


Mathematical models of environmental processes

- Constructs must mimic the involved spatio-temporal dynamics

state	time	space
C	C	C

Partial differential equation (PDE)

$$\frac{\partial \psi}{\partial t} = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}$$

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}$$

Mathematical models of environmental processes

- Constructs must mimic the involved spatio-temporal dynamics

state	time	space	
C	**C**	**C**	Partial differential equation (PDE)

The moment one gives up symbolic manipulation as a major motive for using differential equations, one starts wondering whether one should still keep them as the starting point for numerical modelling; isn't there a less roundabout way to make nature model itself?

– Tommaso Toffoli (1984) –

Mathematical models of environmental processes

- Constructs must mimic the involved spatio-temporal dynamics

state	time	space	
C	C	C	Partial differential equation (PDE)
⋮	⋮	⋮	
D	D	C	**Agent-based model (ABM)**
C	D	D	**Coupled-map lattice (CML)**
D	D	D	**Cellular automaton (CA)**

To PDE or not to PDE?

Omnipresent since

- rigorous mathematical formulation of macroscopic laws
- proven their efficacy
- relatively simple under simplifying assumptions (e.g. homogeneity)
- allow for well-established mathematical analysis
- (less computationally demanding)

though

- mostly approximate solutions obtained by numerical methods
  - truncation and approximation errors
  - stability problems
  - vast number of (complex) methods, often problem specific
- limited to relatively simple initial and boundary conditions
- formulation in heterogeneous media less straightforward
- incompatible with spatio-temporal data in vector or raster format (inherently discrete)

Let's go discrete ...

CA are gaining growing attention from applied scientists

- models mimic life as it is: macroscopic laws may emerge
- CA cells may represent living organisms
- well-suited for modelling processes in heterogeneous media
- irregular boundary and/or initial conditions can be incorporated easily
- no need for numerical approximation methods
- compatible with spatio-temporal data
- may lower the need for expensive, time-consuming laboratory experiments

. . ., but

- lack of quantitative agreement between CA simulations and observed dynamics
- computationally expensive
- lack of a rigorous engineering framework
  - sensitivity analysis
  - model identification
- time steps versus time
- difficult thorough mathematical analysis
- skepticism

Outline

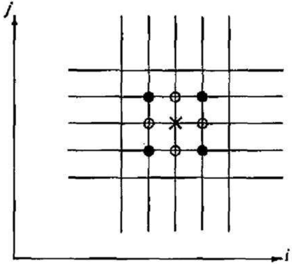
Definition

Cellular automata are simple mathematical idealizations of natural systems. They consist of a lattice of discrete identical sites, each site taking on a finite set of, say, integer values. The values of the sites evolve in discrete time steps according to deterministic rules that specify the value of each site in terms of the values of neighboring sites.

S. Wolfram, 1983

Earliest developments

- Trace back to the work of Turing and von Neumann (early 1900s)
  - Intelligent automata imitating abilities of the human brain (Turing)
  - Quest for a general theory of complicated automata: natural automata and artificial automata (von Neumann)
  - Construction of an artificial, self-reproducing automaton (1940s)
- Stanislaw Ulam suggests von Neumann to use cells as such an automaton's entities: a CA is born (end 1940s)
- von Neumann dies in 1957



Recent resurrection

- Research on CAs fizzled out owing to the decease of its founders and the lack of computing power
- Development of modern computers in the 70s and the growing awareness of the shortcomings of PDEs led to revived interest
  - computer games: ‘Brian’s Brain’, ‘The Game of Life’ (1970), ‘Wa-Tor’ (1984)
  - fundamental research: stability properties by Wolfram, Toffoli, . . .
  - CA-based models: chemistry, ecology, epidemiology, forestry, geography, hydrology, . . .

Cellular automaton

A cellular automaton $\mathscr{C}$ *is a quintuple*

$$\mathscr{C} = \langle \mathcal{T}, S, s, N, \Phi \rangle$$

- $\mathcal{T}$ *is a countably infinite tessellation of a d-dimensional Euclidean space* $\mathbb{R}^d$*, consisting of cells* c_i*,* $i \in \mathbb{N}$*.*
- S *is a finite set of* k *states, often* $S \subset \mathbb{N}$*.*
- s *is the output function, yielding the state value of cell* c_i *at the t-th discrete time step, i.e.* $s(c_i, t)$*.*
- *the neighborhood function* N *maps cell* c_i *to a finite sequence* $N(c_i) = (c_{i_j})_{j=1}^{|N(c_i)|}$*, consisting of* $|N(c_i)|$ *cells* c_{i_j}*.*
- $\Phi = (\phi_i)_{i \in \mathbb{N}}$ *is a family of functions* ϕ_i *governing* c_i*'s dynamics,*

$$s(c_i, t+1) = \phi_i\big(\tilde{s}(N(c_i), t)\big) \, .$$

Order-invariant cellular automaton

An order-invariant cellular automaton (CA) is a CA for which the update of $s(c_i, t)$ is completely independent from the ordering imposed on the elements of $N(c_i)$. Consequently we may write:

$$\phi_i : \mathcal{M}_{|N(c_i)|}(S) \to S\,,$$

where $\mathcal{M}_{|N(c_i)|}$ denotes the set of all multisets of cardinality $|N(c_i)|$ in S, containing at most $|N(c_i)|$ different elements and their respective multiplicity.

Tessellations of $\mathbb{R}^d$

Tessellation

A covering of an infinite geometric space without gaps or overlaps by d-polytopes of one type or a few types

Typically:

- $d = 1$ or $d = 2$ (polygons), infrequently $d = 3$ (polyhedra)
- regular d-polytopes: straightforward implementation
- irregular, convex d-polytopes desirable from an application point of view
- finite tessellations $\mathcal{T}^*$ of a compact subset of $\mathbb{R}^d$

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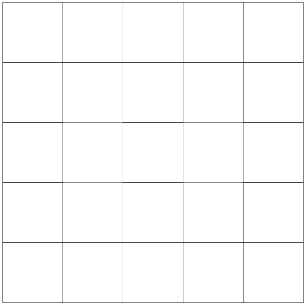

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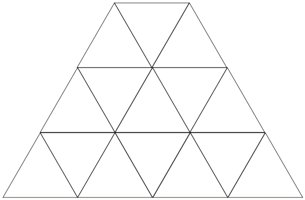

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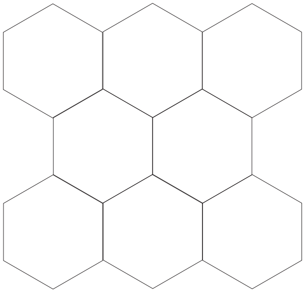

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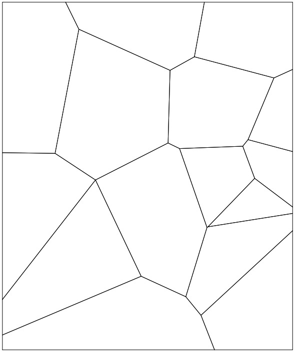

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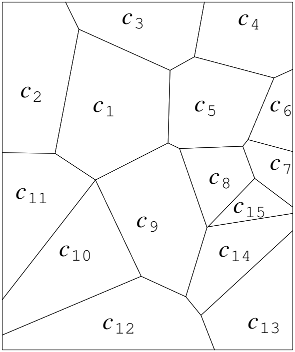
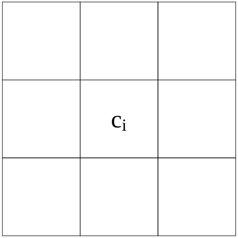


Shortcomings of regular tessellations

1. Restricted number of directions of information flow
2. Directional effects in simulation results
3. Artifacts co-evolved?
4. Limited to processes in media or on surfaces with a simple geometry
5. Incompatible with vector-based spatio-temporal data
6. Neighbourhood not unambiguously defined



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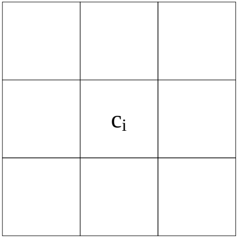

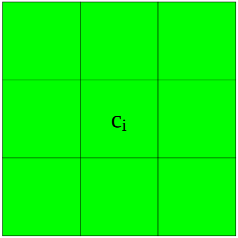

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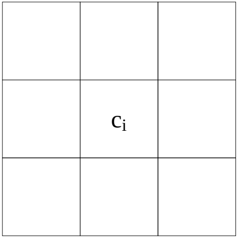

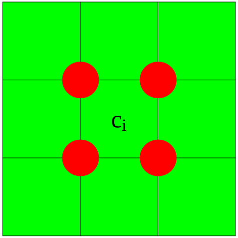

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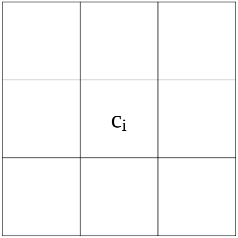

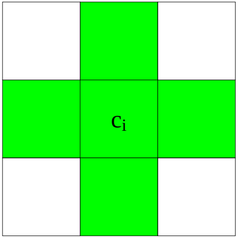

Irregular CA

1. Arbitrary directions of information flow
2. Less directional effects
3. CA dynamics can no longer be attributed to the regularity of the tessellation
4. Phenomena occurring in media with a complex geometry can be described by means of CA-based models by relying on already well-established meshing algorithms used in finite element methods
5. Highly compatible with the vector-based data format originating from spatial observations techniques, such as digital photography, x-ray tomography,. . .
6. Can be employed to devise spatio-temporal models in already existing geographical information software

The neighborhood function

- Aims at selecting those cells that are considered c_i's neighbors
- Falls apart into two operations:
  1. constructing a subset $\mathcal{N}_i \subset \mathcal{T}$ containing c_i's neighbors
  2. imposing an ordering on the elements of $\mathcal{N}_i$

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Moore neighborhood

The Moore neighborhood $\mathcal{N}_i^{\mathcal{M}}$ of c_i contains those $c_j \in \mathcal{T}$ that share a vertex with c_i

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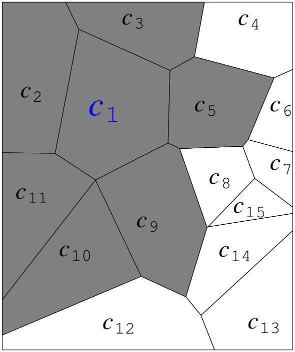

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von Neumann neighborhood

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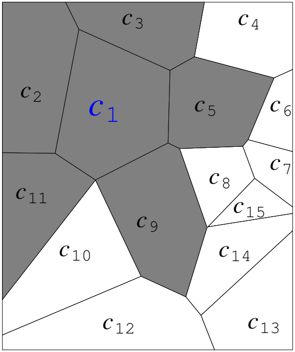

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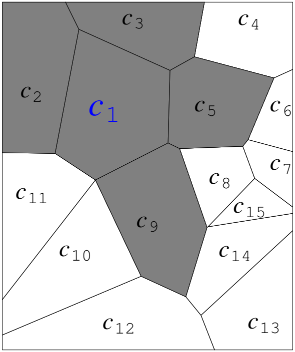
restricted von Neumann neighborhood

The restricted von Neumann neighborhood $\mathcal{N}_i^{\mathcal{V},\nu}$ of c_i contains those $c_j \in \mathcal{T}$ that share an edge with c_i and, additionally, make up at least a prescribed proportion $\nu \in \,]0,1[$ of c_i's circumference



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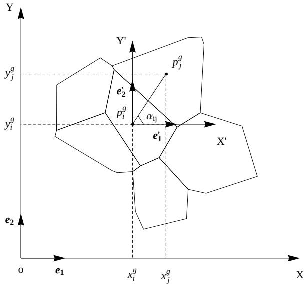
- Sundry possibilities, especially if $d \geq 3$
- If $d = 2$, the angle α_{ij} between the line connecting c_i and c_j's centroids, p_i^g and p_j^g, is a straightforward choice



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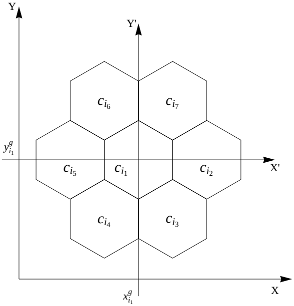

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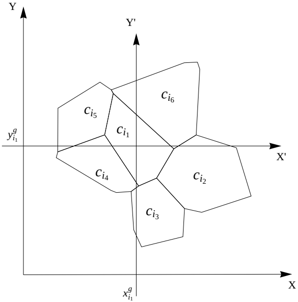


Transition function ϕ_i

Homogeneous cellular automaton

A homogeneous cellular automaton (CA) is a CA for which there exists a $\Theta : \bigcup_{k\in\mathbb{N}} S^k \to S$ such that

$$s(c_i, t+1) = \Theta\big(\tilde{s}\,(N(c_i), t)\big)\,.$$

Order-invariant homogeneous cellular automaton

An order-invariant homogeneous cellular automaton (CA) is a CA for which there exists a $\Psi : \bigcup_{k\in\mathbb{N}} \mathcal{M}_k(S) \to S$ such that

$$s(c_i, t+1) = \Psi\big(\tilde{s}\,(N(c_i), t)\big)\,.$$

Transition function ϕ_i

Two important CA families

1. Totalistic CA
   - order-invariant homogeneous CA for which $S \subset \mathbb{N}$
   - there exists a $\Omega : \mathbb{N} \to S$ such that

$$s(c_i, t+1) = \phi_i\big(\tilde{s}(N(c_i), t)\big) = \Omega\left(\sigma_i\right) \, ,$$

   where $\sigma_i = \sum_{c_j \in N(c_i)} s(c_j, t)$

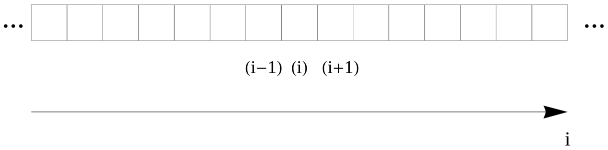
2. Outer-totalistic CA
   - hybrid family between order-variant homogeneous CA and order-invariant CA
   - there exists a $\Omega^* : S \times \mathbb{N} \to S$ such that

$$s(c_i, t+1) = \phi_i\big(\tilde{s}(N(c_i), t)\big) = \Omega^*\left(s(c_i, t), \sigma_i^*\right) \, ,$$

   where $\sigma_i^* = \sigma_i - s(c_i, t)$.

Introduction

- Exemplary order-variant CA family on which most fundamental research is performed
- $d = 1$
- 1-polytopes represented as an 'infinite' sequence of cells



- $S = \{0, 1\}$, such that $k = 2$
- $N(c_i) = (c_{i-1}, c_i, c_{i+1})$ (range $r = 1$)
- Periodic boundary conditions for mimicking $\mathcal{T}$'s infiniteness

Nomenclature

- Θ must be conceived in such a way that every 3-tuple is mapped to either zero (black) or one (white)
- Since $k = 2$, 256 distinct Θ can be listed
- Each rule can be assigned a natural number in $[0, 255]$, *i.e.* a rule number R, according to

$$R = \Theta(1,1,1)\,2^7 + \Theta(1,1,0)\,2^6 + \Theta(1,0,1)\,2^5 + \Theta(1,0,0)\,2^4 + \Theta(0,1,1)\,2^3 + \Theta(0,1,0)\,2^2 + \Theta(0,0,1)\,2^1 + \Theta(0,0,0)\,2^0$$

Nomenclature

Example: Rule 90

$s(c_{i-1}, t)$	$s(c_i, t)$	$s(c_{i+1}, t)$	$s(c_i, t+1)$
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

$$R = \Theta(1,1,1)\,2^7 + \Theta(1,1,0)\,2^6 + \Theta(1,0,1)\,2^5 + \Theta(1,0,0)\,2^4 + \Theta(0,1,1)\,2^3 + \Theta(0,1,0)\,2^2 + \Theta(0,0,1)\,2^1 + \Theta(0,0,0)\,2^0$$

Nomenclature

Example: Rule 90

$s(c_{i-1},t)$	$s(c_i,t)$	$s(c_{i+1},t)$	$s(c_i,t+1)$
1	1	1	**0**
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	0
0	0	1	1
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$$R = 0 \cdot 2^7 + \Theta(1,1,0)\,2^6 + \Theta(1,0,1)\,2^5 + \Theta(1,0,0)\,2^4 + \Theta(0,1,1)\,2^3 + \Theta(0,1,0)\,2^2 + \Theta(0,0,1)\,2^1 + \Theta(0,0,0)\,2^0$$

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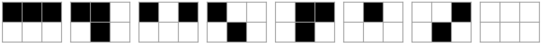
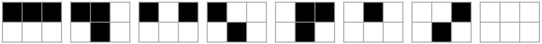
$$
\begin{aligned}
R &= 0 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 \\
&= 90
\end{aligned}
$$

Evolving rule 90

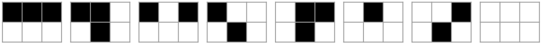
Rule 90: 

Evolving rule 90

Rule 90: 

Suppose s_0 is

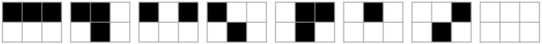
Evolving rule 90

Rule 90: 

Suppose s_0 is

$t = 1$:

Evolving rule 90

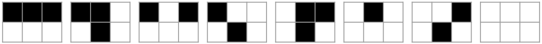
Rule 90: 

Suppose s_0 is

$t = 1$:

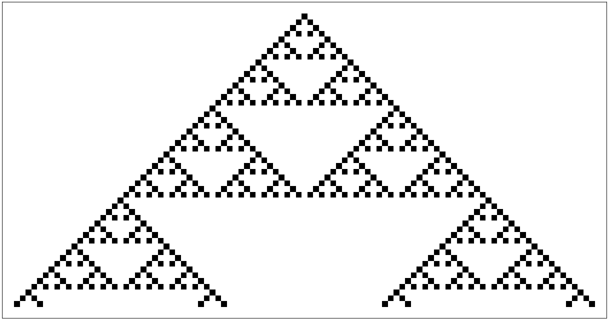
$t = 2$:

Evolving rule 90



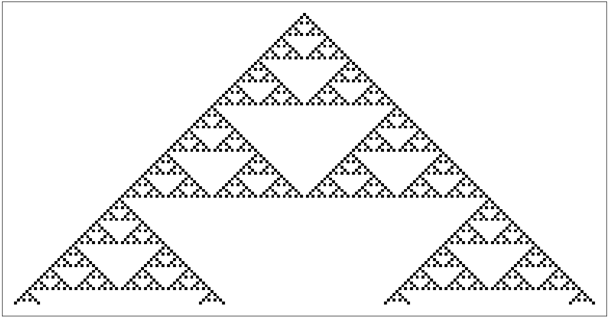
Evolving rule 90

50 time steps



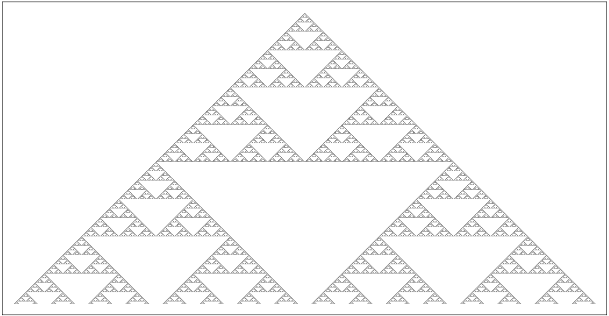
Evolving rule 90

100 time steps



Evolving rule 90

1000 time steps



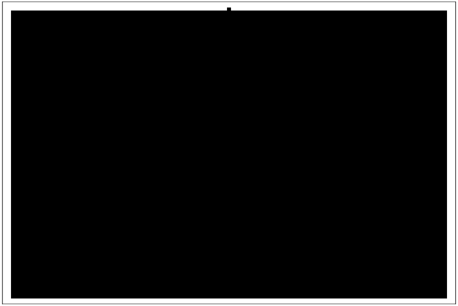
Dynamical properties

- Wolfram distinguished four phenomenological classes based upon visual inspection of the space-time diagrams
- Only three of them can be retrieved in continuous dynamical systems
- It is assumed that this classification transfers to other CA families
- So far, no counterexample overthrowing this hypothesis has been found
- Involves subjectivity
- Quantitative complexity measures have been developed such as the maximum Lyapunov exponent (MLE) λ

Behavioral classes: class 1

CA evolution leads to a homogeneous configuration in which, for example, all $c_i \in \mathcal{T}$ bear the same state (fixed point)

Rule 255: 100 time steps



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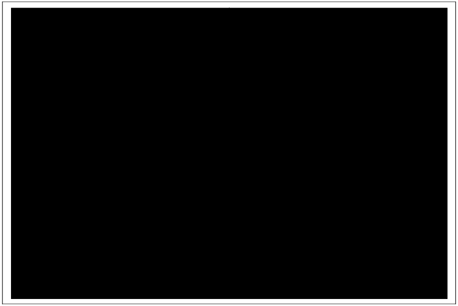

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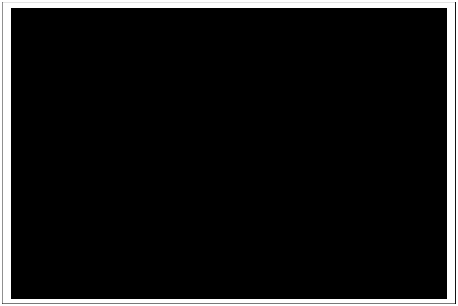
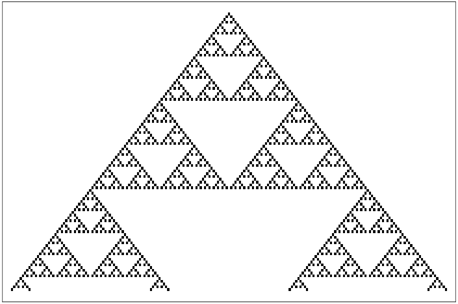


Behavioral classes: class 2

CA evolution leads to a set of stable or periodic structures that are separated and simple (limit cycle)

Rule 90: 100 time steps



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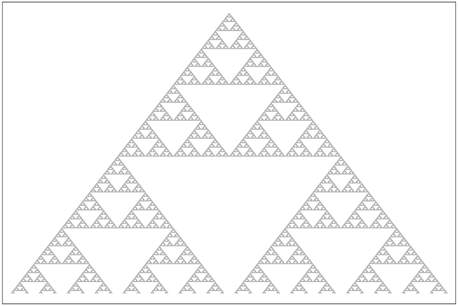


Behavioral classes: fractals

CA rules contained in this class (*e.g.* rule 90) can give rise to fractal patterns

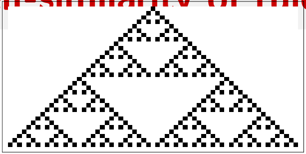
Fractal

A fractal is a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole

Mandelbrot, 1982

- Exhibit self-similarity
- Fractals are omnipresent in nature
    - coast line windings
    - height distribution of a mountain range
    - pigmentation of mammals and molluscs
    - soil structure
- Rule 90: Sierpinski triangle

Self-similarity of rule 90



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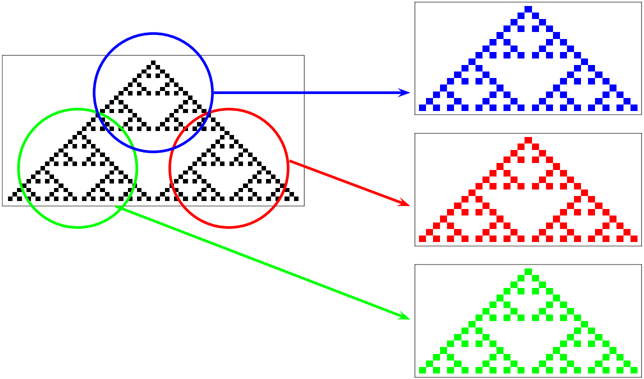

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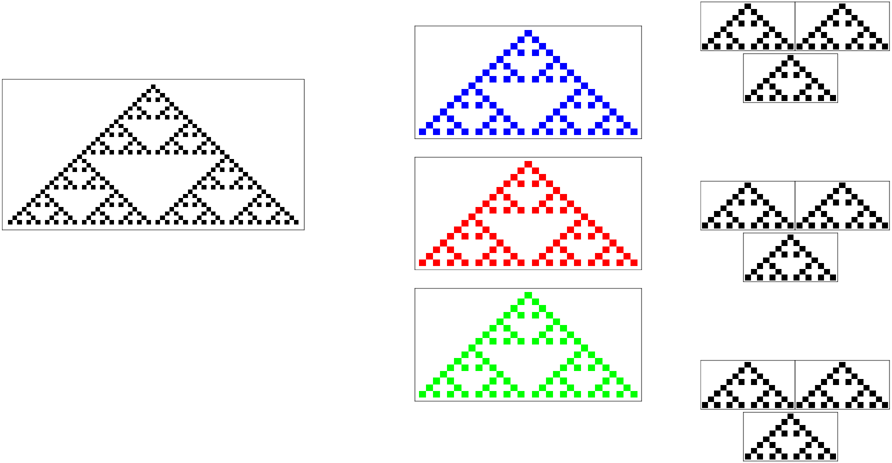

Self-similarity of rule 90: quantification

- Some observations:
  - every triangle is built up from three congruent triangles ($M = 3$)
  - congruent triangles are scaled by a factor $s = 1/2$
- These observations can be exploited to conceive a measure, the fractal dimension $D \in \mathbb{R}^+$, quantifying how completely a fractal fills space

$$D = \frac{\ln(M)}{\ln\left(\frac{1}{s}\right)}$$

- Fractal patterns evolved using rule 90:

$$D = \frac{\ln(3)}{\ln 2} \approx 1.585$$

- line? polygon?

Fractals in nature

- Basically, only a few artificial looking fractals can be evolved by means of CA
- In nature, a wealth of fractal-like structures can be retrieved

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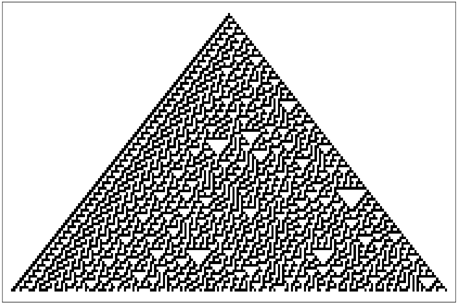
Behavioral classes: class 3

CA evolution leads to a chaotic pattern (strange attractor)

Rule 30: 100 time steps



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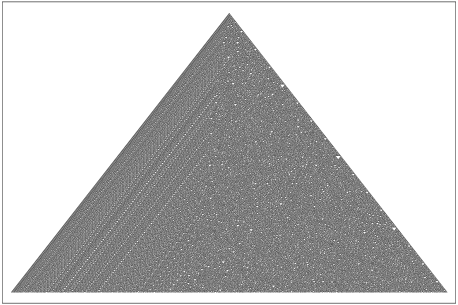
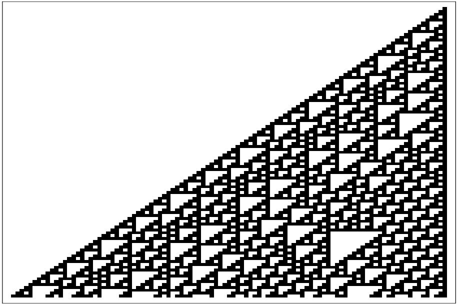


Behavioral classes: class 4

CA evolution leads to complex structures, sometimes long-lived (?)

Rule 110: 100 time steps



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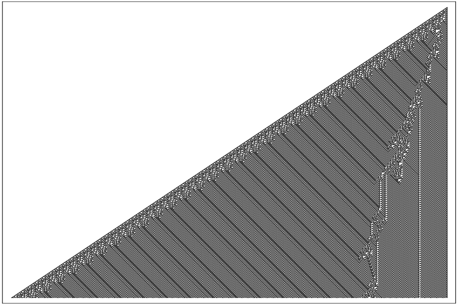
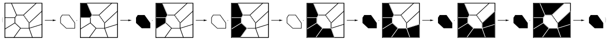


square tessellation

irregular tessellation



Outline

Coupled-map lattice

A coupled-map lattice C is a quintuple

$$C = \langle \mathcal{T}, s, N, \beth \rangle$$

- *$\mathcal{T}$ is a countably infinite tessellation of a d-dimensional Euclidean space $\mathbb{R}^d$, consisting of cells c_i, $i \in \mathbb{N}$.*
- *$s : \mathcal{T} \times \mathbb{N} \to \mathbb{R}$ is the output function, yielding the state value of cell c_i at the t-th discrete time step, i.e. $s(c_i, t)$.*
- *the neighborhood function N maps cell c_i to a finite sequence $N(c_i) = (c_{i_j})_{j=1}^{|N(c_i)|}$, consisting of $|N(c_i)|$ cells c_{i_j}.*
- *$\beth = (\beth_i)_{i \in \mathbb{N}}$ is a family of functions $\beth_i$ governing c_i's dynamics,*

$$s(c_i, t+1) = \beth_i\big(\tilde{s}(N(c_i), t)\big)\,.$$

Outline

SWOT: strengths

- Roundabout way of mathematical modelling is avoided
- No need for numerical approximation methods
- Compatible with spatio-temporal data
- Mean-field assumptions are not a prerequisite
- Irregular boundaries can easily be considered
- Existence of the Planck length and time can be acknowledged

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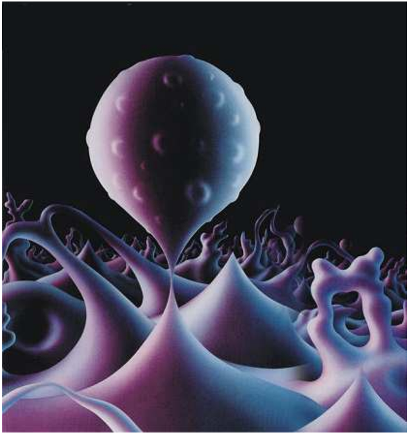
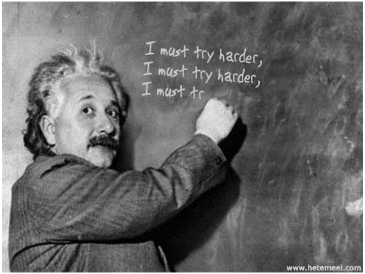


sWOT: weaknesses

- Mathematical analysis is difficult, especially in case of CAs
- Stability properties of CAs are largely unexplored
- Lack of means to assess the quantitative agreement between observations and simulations
- Computationally expensive
- Scepticism



SWOT: threats

- Engineers and mathematicians cling to PDEs
- Unknown, unloved
- Improper use

swOT: opportunities

- Macro: tracking devices
- Meso: femtophotography
- Micro: nanoimaging
- Quantum computing
- Macro: space imaging

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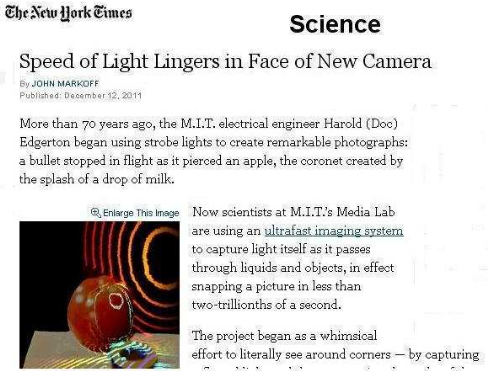

The New York Times

Science

Speed of Light Lingers in Face of New Camera

By JOHN MARKOFF
Published: December 12, 2011

More than 70 years ago, the M.I.T. electrical engineer Harold (Doc) Edgerton began using strobe lights to create remarkable photographs: a bullet stopped in flight as it pierced an apple, the coronet created by the splash of a drop of milk.

Enlarge This Image

Now scientists at M.I.T.'s Media Lab are using an ultrafast imaging system to capture light itself as it passes through liquids and objects, in effect snapping a picture in less than two-trillionths of a second.

The project began as a whimsical effort to literally see around corners — by capturing

SWOT: opportunities

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- Meso: femtophotography
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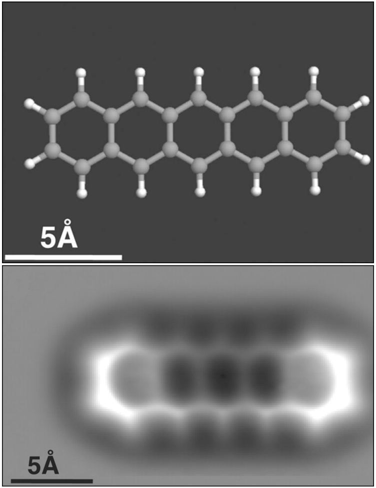
- Quantum computing
- Macro: space imaging



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WETENSCHAP

**

Veelbelovend element voor quantumgeheugen ontdekt

Qubit voor supersnelle computer

Is de computer weer te traag? Met de pas ontdekte Majoranaqubit wil Microsoft supersnelle quantumcomputers bouwen. Martijn van Calmthout

A175 jaar lang wordt er naar gezocht. Maar niemand vond het Majoranafermion. Nochtans voorspelde de geniale Italiaanse wiskundige Ettore Majorana het bestaan ervan al in 1937. Tot Microsoft de Delftse fysicus Leo Kouwenhoven in 2010 een miljoen dollar toestopte om het deeltje te vinden. En hij erin slaagde.

Professor Leo Kouwenhoven maakte de vondst bekend op de jaarvergadering van de Amerikaanse natuurkundige vereniging APS. Het nieuws veroorzaakte een golf van opwinding onder de duizenden aanwezige fysici. Een verslaggever van *Nature* vergeleek het gedrang rond de Nederlander op de wetenschappelijke conferentie met een druk treinstation tijdens de spits.

"Hebben we Majoranafermionen gezien? Ik zou zeggen: een voorzichtig ja", zegt Kouwenhoven aan het eind van zijn presentatie in Boston. Andere natuurkundigen zeiden dat de Delftse metingen niet goed op een andere manier te verklaren zijn dan door de aanwezigheid van een Majorana-achtig deeltje. De resultaten zijn inmiddels gepubliceerd in het tijdschrift *Physical Review Letters*. Kouwenhoven deed zijn vondst niet in een reusachtige deeltjesversneller

Professor Leo Kouwenhoven: 'Een deeltje ontdekken in je eigen laboratorium, mooier kan natuurkunde niet worden.' © SAM RENTMEESTER

Gewone computers werken met bits. Harde eenen en nullen. Maar rekenen gaat

Quantumcomputers werken met qubits. Eentjes die één en nul tegelijk zijn...

Probleem: qubits zijn niet stabiel. In het labo worden ze vlug onstabiel,

Oplossing: De Majorana-qubits komen voor als paar. Omdat de qubits met elkaar

VERDWENEN GENIE

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- Meso: femtophotography
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